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Sum Square Difference Product Prime Labeling of Some Path Related Graphs

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Abstract
Sum square difference product prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with absolute difference of the square of the sum of the labels of the incident vertices and product of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the ***gcin*** of each vertex of degree greater than one is one, then the graph admits sum square difference product prime labeling. Here we identify some path related graphs for sum square difference product prime labeling.
Keywords: Graph labeling, greatest common incidence number, sum square.

1. Introduction

All graphs in this paper are simple, finite, connected and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. In this paper we investigated sum square difference product prime labeling of some path related graphs.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor(gcd) of the labels of the incident edges.

2.Main Results

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges .
Define a bijection f : V(G) → {0,1,2,3,-------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f^*_{ssdppl} : E(G) → set of natural numbers N by $f^*_{ssdppl}(uv) = |\{f(u)+f(v)\}^2 - f(u)f(v)|$.
The induced function f^*_{ssdppl} is said to be sum square difference product prime labeling, if for each vertex of degree at least 2, the greatest common incidence number is 1.

Definition 2.2 A graph which admits sum square difference product prime labeling is called a sum square difference product prime graph.

Theorem 2.1 Path P_n admits sum square difference product prime labeling.
Proof: Let G = P_n and let v_1,v_2,----------------,v_n are the vertices of G
Here |V(G)| = n and |E(G)| = n-1
Define a function f : V → {0,1,2,3,----------------,n-1 } by

$$f(v_i) = i-1 , \quad i = 1,2,------,n$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$$f^*_{ssdppl}(v_i\, v_{i+1}) = 3i^2-3i+1, \qquad i = 1,2,------------,n-1$$

Clearly f^*_{ssdppl} is an injection.

$$\begin{aligned}
\textit{gcin} \text{ of } (v_{i+1}) &= \text{gcd of } \{ f^*_{ssdppl}(v_i\, v_{i+1}),\ f^*_{ssdppl}(v_{i+1}\, v_{i+2}) \} \\
&= \text{gcd of } \{ 3i^2-3i+1,\ 3i^2+3i+1\} \\
&= \text{gcd of } \{6i ,\ 3i^2-3i+1\} \\
&= \text{gcd of } \{3i ,\ 3i^2-3i+1\} \\
&= \text{gcd of } \{3i ,\ 3i(i-1)+1\}
\end{aligned}$$

$= 1, \quad i = 1,2,\text{------------},n-2$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence P_n , admits sum square difference product prime labeling. ■

Theorem 2.2 Middle graph of Path P_n admits sum square difference product prime labeling.
Proof: Let G =M(P_n) and let $v_1,v_2,\text{----------------},v_{2n-1}$ are the vertices of G
Here |V(G)| = 2n-1 and |E(G)| = 3n-4
Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},2n-2\}$ by

$$f(v_i) = i-1 , i = 1,2,\text{------},2n-1$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\ v_{i+1}) = 3i^2-3i+1, i = 1,2,\text{--------},2n-2$

$f^*_{ssdppl}(v_{2i}\ v_{2i+2}) = 12i^2+1, \quad i = 1,2,\text{--------},n-2$

Clearly f^*_{ssdppl} is an injection.

gcin of (v_{i+1}) $= 1, \quad i = 1,2,\text{--------},2n-3$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence M(P_n), admits sum square difference product prime labeling. ■

Theorem 2.3 Total graph of Path P_n admits sum square difference product prime labeling.
Proof: Let G =T(P_n) and let $v_1,v_2,\text{----------------},v_{2n-1}$ are the vertices of G
Here |V(G)| = 2n-1 and |E(G)| = 4n-5
Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},2n-2\}$ by

$$f(v_i) = i-1 , i = 1,2,\text{------},2n-1$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\ v_{i+1}) = 3i^2-3i+1, \quad i = 1,2,\text{-----},2n-2$

$f^*_{ssdppl}(v_{2i}\ v_{2i+2}) = 12i^2+1, \quad i = 1,2,\text{-------},n-2$

$f^*_{ssdppl}(v_{2i-1}\ v_{2i+1}) = 12i^2-12i+4, i = 1,2,\text{-------},n-1$

Clearly f^*_{ssdppl} is an injection.

gcin of (v_1) $= 1,$
gcin of (v_{i+1}) $= 1, \quad i = 1,2\text{------},2n-3$
gcin of (v_{2n-1}) $= \text{gcd of } \{f^*_{ssdppl}(v_{2n-1}\ v_{2n-2}), f^*_{ssdppl}(v_{2n-1}\ v_{2n-3})\}$
$= \text{gcd of } \{12n^2-30n+19, 12n^2-36n+28\}$
$= \text{gcd of } \{6n-9, 12n^2-36n+28\}$
$= \text{gcd of } \{6n-9, (6n-9)(2n-3)+1\}$
$= 1$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence T(P_n), admits sum square difference product prime labeling. ■

Theorem 2.4 2-tuple graph of Path P_n admits sum square difference product prime labeling.
Proof: Let G =T^2(P_n) and let $v_1,v_2,\text{----------------},v_{2n}$ are the vertices of G
Here |V(G)| = 2n and |E(G)| = 3n-2
Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},2n-1\}$ by
$f(v_i) = i-1 , i = 1,2,\text{------},2n$
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\ v_{i+1}) = 3i^2-3i+1, i = 1,2,\text{--------},2n-1$

$f^*_{ssdppl}(v_i\ v_{2n+1-i}) = (2n-1)^2 – (2n-i)(i-1),$
$i = 1,2,\text{----------},n-1$

Clearly f^*_{cdpl} is an injection.

gcin of (v_1) $= 1,$
gcin of (v_{i+1}) $= 1, \quad i = 1,2,\text{--------},2n-2$
gcin of (v_{2n}) $= \text{gcd of } \{f^*_{ssdppl}(v_1\ v_{2n}), f^*_{ssdppl}(v_{2n-1}\ v_{2n})\}$
$= \text{gcd of } \{(2n-1)^2, 12n^2-18n+7\}$
$= \text{gcd of } \{(2n-1), 12n^2-18n+7\}$
$= \text{gcd of } \{(2n-1), (2n-1)(6n-6)+1\}$
$= 1$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $T^2(P_n)$, admits sum square difference product prime labeling . ■

Theorem 2.5 P_n^2 admits sum square difference product prime labeling .
Proof: Let G = P_n^2 and let $v_1,v_2,\text{----------------},v_n$ are the vertices of G
Here |V(G)| = n and |E(G)| = 2n-3
Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},n-1\}$ by

$$f(v_i) = i-1 , i = 1,2,\text{------},n$$

Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\ v_{i+1}) = 3i^2-3i+1, i = 1,2,\text{----------},n-1$

$f^*_{ssdppl}(v_i\ v_{i+2}) = 3i^2+1, \quad i = 1,2,\text{----------},n-2$

Clearly f^*_{ssdppl} is an injection.

gcin of (v_1) $= 1,$
gcin of (v_{i+1}) $= 1, \quad i = 1,2,\text{------------},n-2$
gcin of (v_n) $= \text{gcd of } \{f^*_{ssdppl}(v_{n-1}\ v_n), f^*_{ssdppl}(v_{n-2}\ v_n)\}$

$= \gcd$ of $\{3n^2-12n+13, 3n^2-9n+7\}$
$= \gcd$ of $\{(3n-6), 3n^2-12n+13\}$
$= \gcd$ of $\{(3n-6), (3n-6)(n-2)+1\}$
$= 1$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence P_n^2, admits sum square difference product prime labeling. ■

Theorem 2.6 Duplicate graph of path P_n admits sum square difference product prime labeling.
Proof: Let $G = D(P_n)$ and let v_1, v_2,----------------,v_{2n} are the vertices of G
Here $|V(G)| = 2n$ and $|E(G)| = 2n-2$
Define a function $f : V \rightarrow \{0,1,2,3,$----------------$,2n-1\}$ by
$f(v_i) = i-1$, i = 1,2,------,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling
f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\ v_{i+1}) = 3i^2-3i+1$, i = 1,2,------------,n-1
$f^*_{ssdppl}(v_{n+i}\ v_{n+i+1}) = (2n+2i-1)^2-(n+i)(n+i-1)$, i = 1,2,------------,n-1

Clearly f^*_{ssdppl} is an injection.
gcin of $(v_{i+1}) = 1$, i = 1,2,------------,n-2
gcin of $(v_{n+i+1}) = 1$, i = 1,2,------------,n-2
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $D(P_n)$, sum square difference product prime labeling. ■

Theorem 2.7 Strong Duplicate graph of path P_n admits sum square difference product prime labeling., when $n \not\equiv 0 \pmod 7$
Proof: Let $G = SD(P_n)$ and let v_1, v_2,----------------,v_{2n} are the vertices of G
Here $|V(G)| = 2n$ and $|E(G)| = 3n-2$
Define a function $f : V \rightarrow \{0,1,2,3,$----------------$,2n-1\}$ by
$f(v_i) = i-1$, i = 1,2,------,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling
f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\ v_{i+1}) = 3i^2-3i+1$, i = 1,2,------------,2n-1
$f^*_{ssdppl}(v_{2i-1}\ v_{2i+2}) = 12i^2-6i+3$, i = 1,2,------------,n-1

Clearly f^*_{ssdppl} is an injection.
gcin of $(v_1) = 1$
gcin of $(v_{i+1}) = 1$, i = 1,2,------------,2n-2
gcin of $(v_{2n}) = \gcd$ of $\{f^*_{ssdppl}(v_{2n-1}\ v_{2n}),\ f^*_{ssdppl}(v_{2n-3}\ v_{2n})\}$
$= \gcd$ of $\{12n^2-18n+7, 12n^2-30n+21\}$
$= 1$

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $SD(P_n)$, admits sum square difference product prime labeling. ■

Theorem 2.8 Shadow graph of path P_n admits sum square difference product prime labeling, when $(n+1) \not\equiv 0 \pmod 7$
Proof: Let $G = D_2(P_n)$ and let v_1, v_2,----------------,v_{2n} are the vertices of G
Here $|V(G)| = 2n$ and $|E(G)| = 4n-4$
Define a function $f : V \rightarrow \{0,1,2,3,$----------------$,2n-1\}$ by
$f(v_i) = i-1$, i = 1,2,------,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling
f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_{2i-1}\ v_{2i+1}) = 12i^2-12i+4$, i = 1,2,------------,n-1
$f^*_{ssdppl}(v_{2i}\ v_{2i+2}) = 12i^2+1$, i = 1,2,------------,n-1
$f^*_{ssdppl}(v_{2i-1}\ v_{2i+2}) = 12i^2-6i+3$, i = 1,2,------------,n-1
$f^*_{ssdppl}(v_{2i}\ v_{2i+1}) = 12i^2-6i+1$, i = 1,2,------------,n-1

Clearly f^*_{ssdppl} is an injection.
gcin of $(v_1) = 1$
gcin of $(v_2) = 1$
gcin of $(v_{2i+1}) = \gcd$ of $\{f^*_{ssdppl}(v_{2i-1}\ v_{2i+1}),\ f^*_{ssdppl}(v_{2i}\ v_{2i+1})\}$
$= \gcd$ of $\{12i^2-12i+4, 12i^2-6i+1\}$
$= \gcd$ of $\{12i^2-12i+4, 6i-3\}$
$= \gcd$ of $\{(6i-3)(2i-1)+1, 6i-3\}$
$= 1$, i = 1,2,------------,n-1
gcin of $(v_{2i+2}) = \gcd$ of $\{f^*_{ssdppl}(v_{2i+4}\ v_{2i+2}),\ f^*_{ssdppl}(v_{2i}\ v_{2i+2})\}$
$= \gcd$ of $\{12i^2 + 1, 12i^2+24i+13\}$
$= \gcd$ of $\{12i^2 + 1, 24i+12\}$
$= \gcd$ of $\{(12i^2 + 1, 6i+3\}$
$= \gcd$ of $\{4, 6i+3\} = \gcd$ of $\{2, 6i+3\}$
$= 1$, i = 1,2,------------,n-2
gcin of $(v_{2n}) = \gcd$ of $\{f^*_{ssdppl}(v_{2n-3}\ v_{2n}),\ f^*_{ssdppl}(v_{2n-2}\ v_{2n})\}$
$= \gcd$ of $\{12n^2-30n+21, 12n^2-24n+13\}$
$= \gcd$ of $\{12n^2-30n+21, 6n-8\}$
$= \gcd$ of $\{12n^2-30n+21, 3n-4\}$

= gcd of { n+1, 3n-4}
= gcd of { n+1, n-6}= gcd of { 7, n-6}
= 1

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $D_2(P_n)$, admits sum square difference product prime labeling. ■

Theorem 2.9 Strong shadow graph of path P_n admits sum square difference product prime labeling.

Proof: Let G = S{$D_2(P_n)$} and let v_1, v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 5n-4

Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_{2i-1}\, v_{2i+1}) = 12i^2-12i+4$, i = 1,2,------------,n-1

$f^*_{ssdppl}(v_{2i}\, v_{2i+2}) = 12i^2+1$, i = 1,2,------------,n-1

$f^*_{ssdppl}(v_{2i-1}\, v_{2i+2}) = 12i^2-6i+3$, i = 1,2,------------,n-1

$f^*_{ssdppl}(v_{2i}\, v_{2i+1}) = 12i^2-6i+1$, i = 1,2,------------,n-1

$f^*_{ssdppl}(v_{2i}\, v_{2i-1}) = 12i^2-18i+7$, i = 1,2,------------,n

Clearly f^*_{ssdppl} is an injection.

gcin of (v_1) = 1

gcin of (v_{i+1}) = 1, i = 1,2,------------,2n-2

gcin of (v_{2n}) = gcd of {$f^*_{ssdppl}(v_{2n-1}\, v_{2n})$, $f^*_{ssdppl}(v_{2n-2}\, v_{2n})$ }
= gcd of { $12n^2$-24n+13, $12n^2$-18n+7}
= gcd of { $12n^2$-24n+13, 6n-6}
= gcd of { (6n-6)(2n-2)+1, 6n-6}
= 1

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence S{$D_2(P_n)$}, admits sum square difference product prime labeling. ■

Theorem 2.10 Z graph of path P_n admits sum square difference product prime labeling.

Proof: Let G = Z(P_n) and let v_1, v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n-3

Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{cdpl} is defined as follows

$f^*_{cdpl}(v_{2i-1}\, v_{2i+1}) = 12i^2-12i+4$, i = 1,2,------------,n-1

$f^*_{cdpl}(v_{2i}\, v_{2i+2}) = 12i^2 + 1$, i = 1,2,------------,n-1

$f^*_{cdpl}(v_{2i}\, v_{2i+1}) = 12i^2 -6i+1$, i = 1,2,------------,n-1

Clearly f^*_{ssdppl} is an injection.

gcin of (v_{2i+1}) = gcd of {$f^*_{ssdppl}(v_{2i-1}\, v_{2i+1})$, $f^*_{ssdppl}(v_{2i}\, v_{2i+1})$ }
= gcd of { $12i^2$-12i+4, $12i^2$-6i+1}
= gcd of { $12i^2$-12i+4, 6i-3}
= gcd of { (6i-3)(2i-1)+1, 6i-3}
= 1, i = 1,2,------------,n-1

gcin of (v_{2i}) = gcd of {$f^*_{ssdppl}(v_{2i}\, v_{2i+2})$, $f^*_{ssdppl}(v_{2i}\, v_{2i+1})$ }
= gcd of { $12i^2$+1, $12i^2$-6i+1}
= gcd of { $12i^2$-6i+1, 6i}
= gcd of { (2i-1)(6i)+1, 6i}
= 1, i = 1,2,------------,n-1

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence Z(P_n), admits sum square difference product prime labeling. ■

4. Conclusions

In this paper we proved that some path related graphs admit sum square difference product prime labeling. Here shadow graph and duplicate graph does not admit sum square difference product prime labeling according to our pattern of labeling.

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